FORMULAE

[FORMULA - A special type of equation that shows the relationship between different variables.]

I. Number Theory

1 Rational Number (between two given rational numbers ‘$\boldsymbol{a}$’ and ‘$\boldsymbol{b}$’) = $a + \frac{n(b-a)}{N}$, where $\boldsymbol{n}$ is the sequence of the rational number and $\boldsymbol{N}$ is the total number of rational numbers which we need to find.

2 **Euclid’s Division Lemma** $a = bq + r$, where a is dividend, b is divisor, q is quotient and r is remainder.

3 The Fundamental Theorem of Arithmetic $LCM \times HCF = Product\ of\ the\ numbers$

4 The Result of the Fundamental Theorem of Arithmetic

$$LCM\ (p,q,r) = \frac{p.q.r.HCF(p,q,r)}{HCF(p,q).HCF(q,r).HCF(p,r)}$$

$$HCF\ (p,q,r) = \frac{p.q.r.LCM(p,q,r)}{LCM(p,q).LCM(q,r).LCM(p,r)}$$

II. Algebra

1 $(a+b)^2 = a^2 + 2ab + b^2$

2 $(a-b)^2 = a^2 - 2ab + b^2$

3 $(a+b)(a-b) = a^2 - b^2$

4 $(a+b)^2 + (a-b)^2 = 2\,(a^2 + b^2)$

5 $(a+b)^2 - (a-b)^2 = 4ab$

6 $a^2 + b^2 = (a+b)^2 - 2ab = (a-b)^2 + 2ab$

7 $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$

8 $\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$

9 $(x+a)(x+b) = x^2 + (a+b)x + ab$

10 $(x-a)(x+b) = x^2 - (a-b)x - ab$

11 $(x+a)(x-b) = x^2 + (a-b)x - ab$

12 $(x-a)(x-b) = x^2 - (a+b)x + ab$

13 $(a+b)^3 = a^3 + 3ab(a+b) + b^3 = a^3 + 3a^2b + 3ab^2 + b^3$

14 $(a-b)^3 = a^3 - 3ab(a-b) - b^3 = a^3 - 3a^2b + 3ab^2 - b^3$

15 $(a+b)^3 + (a-b)^3 = 2\,a^3 + 6\,ab^2$

16 $(a+b)^3 - (a-b)^3 = 2\,b^3 + 6\,a^2b$

17 $a^3 + b^3 = (a+b)(a^2 - ab + b^2) = (a+b)^3 - 3a^2b - 3ab^2$
$= (a+b)^3 - 3ab(a+b)$

18 $a^3 - b^3 = (a-b)(a^2 + ab + b^2) = (a-b)^3 + 3a^2b - 3ab^2$
$= (a-b)^3 - 3ab(a-b)$

19 $\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$

20 $\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$

21 $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right)^3 - 3x - 3 \times \frac{1}{x} = \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - 1\right)$

22 $x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 + 3\left(x + \frac{1}{x}\right) = \left(x - \frac{1}{x}\right)^3 + 3x - 3 \times \frac{1}{x} = \left(x - \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + 1\right)$

23 $(\pm a \pm b \pm c)^2 = a^2 + b^2 + c^2 \pm 2ab \pm 2bc \pm 2ca$

24 $(a + b + c)^3 = a^3 + b^3 + c^3 + 3ab(a + b) + 3bc(b + c) + 3ca(c + a) + 6abc$

25 $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
$= \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$

26 $a^4 - b^4 = (a^2 + b^2)(a + b)(a - b)$

27 $a^m \times a^n = a^{m+n}$

28 $a^m \div a^n = a^{m-n}$

29 $(a^m)^n = a^{mn}$

30 $a^m \times b^m = (a \times b)^m$

31 $a^m \div b^m = (a \div b)^m$

32 $a^{-m} = \frac{1}{a^m}$

33 $\sqrt[m]{a} = a^{1/m}$

34 $a^0 = 1$

35 $log_a m + log_a n = log_a(m \times n)$

36 $log_a m - log_a n = log_a(m \div n)$

37 $log_a m^n = n log_a m$

38 $log_a a = 1$

39 $log_b a = \frac{1}{log_a b} = \frac{log_e a}{log_e b}$

40 $log_b a = log_{c_1} a \times log_{c_2} c_1 \ldots\ldots\ldots\ldots \times log_{c_n} c_{n-1} \times log_b c_n$

41 *Remainder Theorem* - If $p(x)$ is any polynomial of degree greater than or equal to 1 and $p(x)$ is divided by the linear polynomial $x - a$, then the remainder is $p(a)$.

42 *Factor Theorem* – $(x - a)$ is a factor of the polynomial $p(x)$, if $p(a) = 0$. Also, if $x - a$ is a factor of $p(x)$, then $p(a) = 0$.

43 *Quadratic Formula* $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$,
where the values of x are the roots of the quadratic equation $ax^2 + bx + c = 0$ *and* $D(= b^2 - 4ac)$ *is the Discriment.*

(a) If D < 0, then the roots are non-real.
(b) If D = 0, then the roots are real and equal.
(c) If D > 0, then the roots are real and unequal.

If the roots of the quadratic equation $ax^2 + bx + c = 0$ are α and β, then

$sum\ of\ roots = \alpha + \beta = -\frac{b}{a} = -\frac{Coefficient\ of\ x}{Coefficient\ of\ x^2}$ and

$product\ of\ roots = \alpha . \beta = \frac{c}{a} = \frac{Constant\ term}{Coefficient\ of\ x^2}$

For any three sets A, B & C and Universal set ξ (xi), we have

44 $A \times (B \cup C) = (A \times B) \cup (A \times C)$

45 $A \times (B \cap C) = (A \times B) \cap (A \times C)$

46 $A \times (B - C) = (A \times B) - (A \times C)$

47 $A \times B = B \times A \Leftrightarrow A = B$

48 $(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$

49 $A \times (B' \cup C')' = (A \times B) \cap (A \times C)$

50 $A \times (B' \cap C')' = (A \times B) \cup (A \times C)$

51 $A \times B = A \times C \Rightarrow B = C$

52 *Idempotent laws* $A \cup A = A$ *and* $A \cap A = A$

53 *Identity laws* $A \cup \phi = A$ *and* $A \cap \xi = A$

54 *Commutative laws* $A \cup B = B \cup A$ *and* $A \cap B = B \cap A$

55 *Associative laws* $(A \cup B) \cup C = A \cup (B \cup C)$ and $(A \cap B) \cap C = A \cap (B \cap C)$

56 *Distributive laws* $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

57 *De' Morgan's laws* $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$

58 *Complementary laws* $A \cup A' = \xi$ and $A \cap A' = \phi$

59 Let S be the sample space associated with a random experiment. A set of events $A_1, A_2, A_3, \ldots\ldots\ldots, A_n$ is said to form a set of mutually exclusive and exhaustive system of events if

(i) $A_1 \cup A_2 \cup A_3 \cup \ldots\ldots\ldots, \cup A_n = S$

(ii) $A_i \cap A_j = \emptyset$ for $i \neq j$

60 Probability Function: - Let S = $\{w_1, w_2, \ldots\ldots\ldots, w_n\}$ be the sample space associated with a random experiment. Then a function P which assigns every event $A \subset S$ to a unique non-negative real number P(A) is called the probability function if the following axioms hold:

A – 1 $0 \leq P(w_i) \leq 1$ for all $w_i \in S$

A – 2 P (S) = 1, i.e. $P(w_1) + P(w_2) + \ldots\ldots + P(w_n) = 1$.

A – 3 For any event $A \subset S$, P (A) = $\sum P(w_k)$, the number $P(w_k)$ is called the probability of elementary event w_k.

61 Probability of an event: - If there are n elementary events associated with a random experiment and m of them are favourable to an event A, then the probability of occurrence of A is defined as:

$$P(A) = \frac{m}{n} = \frac{Favourable\ number\ of\ elementary\ events}{Total\ number\ of\ elementary\ events}$$

The odds in favour of occurrence of the event A are defined by: $m : (n - m)$

The odds against the occurrence of A are defined by : $(n - m) : m$.

The probability of non-occurrence of A is given by $P(\bar{A}) = 1 - P(A)$

62 If A and B are two events associated with a random experiment, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

If A and B are mutually exclusive events, then

$$P(A \cup B) = P(A) + P(B).$$

63 If A, B, C are three events associated with a random experiment, then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

64 If A and B are two events associated with a random experiment, then

(i) $P(\bar{A} \cap B) = P(B) - P(A \cap B)$, i.e. probability of occurrence of B only

(ii) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$, i.e. probability of occurrence of A only

(iii) Probability of occurrence of exactly one of A and B
$= P(A) + P(B) - 2P(A \cap B) = P(A \cup B) - P(A \cap B)$

If n is a natural number and r is a non-negative integer such that $0 \leq r \leq n$, then

65 $^nC_r = \dfrac{n!}{(n-r)!\,r!}$

66 $^nC_r \times r! = {}^nP_r$

67 $^nC_r = {}^nC_{n-r}$

68 $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

69 $^nC_r = \frac{n}{r}.{}^{n-1}C_{r-1} = \frac{n}{r} \times \frac{(n-1)}{(r-1)}.{}^{n-2}C_{r-2} = \ldots\ldots = \frac{n}{r} \times \frac{(n-1)}{(r-1)} \times \frac{(n-2)}{(r-2)} \times \ldots \times \frac{n-(r-1)}{1}$

70 $^nC_x = {}^nC_y \Rightarrow x = y$ **OR** $x + y = n$

71 If n is an even natural number, then the greatest among

$$^nC_0, {}^nC_1, {}^nC_2, \ldots\ldots, {}^nC_n \text{ is } {}^nC_{\frac{n}{2}}$$

72 If n is an odd natural number, then the greatest among

$$^nC_0, {}^nC_1, {}^nC_2, \ldots\ldots, {}^nC_n \text{ is } {}^nC_{\frac{n-1}{2}} \text{ or } {}^nC_{\frac{n+1}{2}}$$

73 The number of ways of selecting **r** items or objects from a group of n distinct items or objects is

$$\frac{n!}{(n-r)!\,r!} = {}^nC_r$$

74 Binomial Theorem: - If x and a are real numbers, then for all n ∈ N, we have

$$(x+a)^n = {}^nC_0 x^n a^0 + {}^nC_1 x^{n-1} a^1 + {}^nC_2 x^{n-2} a^2 + \ldots.. + {}^nC_r x^{n-r} a^r + \ldots.. + {}^nC_{n-1} x^1 a^{n-1} + {}^nC_n x^0 a^n$$

i.e., $$(x+a)^n = \sum_{r=0}^{n} {}^nC_r x^{n-r} a^r$$

75 The sum S_n of n terms of an A.P. with first term a and common difference d is given by

$S_n = \frac{n}{2}[2a + (n-1)d]$ or $S_n = \frac{n}{2}[a + l]$, where l = last term = $a + (n-1)d$

76 If the sum S_n of n terms of a sequence is given, then n^{th} term a_n of the sequence can be determined by using the formula $a_n = S_n - S_{n-1}$

77 If n numbers $A_1, A_2, \ldots\ldots A_n$ are inserted between two given numbers a and **b** such that $a, A_1, A_2, \ldots\ldots A_n, b$ is an arithmetic progression, then $A_1, A_2, \ldots\ldots A_n$ are known as n arithmetic means between a and **b** and the common difference of the A. P. is $d = \frac{b-a}{n+1}$.

Also, $A_1, A_2, \ldots\ldots A_n = n\left(\frac{a+b}{2}\right)$.

78 Three numbers a, b, c are in A. P. iff (if and only if) $2b = a + c$.

79 The Arithmetic Mean of a and b is $\frac{a+b}{2}$.

80 The n^{th} term of a Geometric Progression with first term a and common ratio r is given by

$$a_n = ar^{n-1}$$

81 If the sum of n terms of a G. P. with first term a and common ratio is given by

$$S_n = a\left(\frac{r^n - 1}{r-1}\right) \quad for\ \boldsymbol{r} > 1$$

$$S_n = a\left(\frac{1-r^n}{1-r}\right) \quad for\ \boldsymbol{r} < 1$$

$$S_n = n, \text{ if } r = 1$$

Also, $S_n = \frac{a - lr}{1-r}$ or $S_n = \frac{lr - a}{r-1}$, where l is the last term.

82 Let a and b be two given numbers. If n numbers $G_1, G_2, \ldots\ldots G_n$ are inserted between a and b such that the sequence $a, G_1, G_2, \ldots\ldots G_n, b$ is a G. P., then the numbers $G_1, G_2, \ldots\ldots G_n$ are known as n geometric means between a and b. The common ratio of the G.P. is given by $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

83 The Geometric Mean of a and b is given by $\sqrt{ab}$.

84 If A and G are respectively Arithmetic and Geometric Means between two positive numbers a and b, then

(i) A > G.

(ii) The quadratic equation having a and b as its roots is $x^2 - 2Ax + G^2 = 0$.

(iii) $a : b = \left(A + \sqrt{A^2 - G^2}\right) : \left(A - \sqrt{A^2 - G^2}\right)$.

85 If AM and GM between two numbers are in the ratio m : n, then the numbers are in the ratio $\left(m + \sqrt{m^2 - n^2}\right) : \left(m - \sqrt{m^2 - n^2}\right)$.

86 Three numbers a, b, c are in G. P. iff (if and only if) $b^2 = ac$.

For any n ∈ N, we have

87 $$\sum_{k=1}^{n} k = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

88 $$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

89 $$\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

90 $$\sum_{k=1}^{n} k^4 = 1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

In a series $a_1 + a_2 + a_3 + a_4 + \cdots + a_n + \cdots$, (for n ∈ N)

91 (i) If the differences $a_2 - a_1, a_3 - a_4, a_4 - a_3, \ldots \ldots$ are in A. P., then the n[th] term is given by $a_n = an^2 + \ an + c$, where a, b and c are constants.
(ii) If the differences $a_2 - a_1, a_3 - a_4, a_4 - a_3, \ldots \ldots$ are in G. P., with common ratio r, then $a_n = ar^{n-1} + b_n + c$, where a, b and c are constants.

92 Let $A = [a_{ij}]$ be an $m \times n$ matrix. Then, the transpose of A, denoted by A^T, is an $n \times m$
Matrix such that $(A^T)_{ij} = a_{ji}$ for all $i = 1, 2, \ldots. m;\ j = 1, 2, \ldots., n$.
Following are the properties of transpose of a matrix:
(i) $(A^T)^T = A$ **(ii)** $(A+B)^T = A^T + B^T$ **(iii)** $(kA)^T = kA^T$ **(iv)** $(AB)^T = B^T A^T$
(v) $(ABC)^T = C^T B^T A^T$

93 If A is a non-singular matrix, then $A^{-1} = \frac{1}{|A|}(adj\ A)$

94 If A and B are two invertible matrices of the same order, then $(AB)^{-1} = B^{-1}A^{-1}$

95 A system $AX = B$ of n linear equations in n equations has a unique solution given by $X = A^{-1}B$, if $|A| \neq 0$.
If $|A| = 0$ and $(adj\ A)B = 0$, then the system is consistent and has infinitely many solutions. If $|A| = 0$ and $(adj\ A)B \neq 0$, then the system is inconsistent.

96 A homogeneous system of n linear equations in n unknowns is expressible in the form AX =O. If $|A| \neq 0$, then AX =O has unique solution X = 0, i.e. $x_1 = x_2 = \cdots = x_n = 0$. This solution is called the Trivial solution. If $|A| = 0$, then AX =O has infinitely many solutions.

If $\vec{a}$ and $\vec{b}$ are two non-zero vectors inclined at an angle θ, then

97 ***Scalar Product or Dot Product of two vectors*** – Let $\vec{a}$ and $\vec{b}$ be two vectors and let θ be the angle between them. Then, the scalar product or dot product of $\vec{a}$ and $\vec{b}$, denoted by $\vec{a}.\vec{b}$, is defined as

$$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta = \text{ab}\ \cos\theta.$$

Therefore $\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} \Rightarrow \theta = \cos^{-1}\left(\dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}\right)$

98 Projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}\,.\vec{b}}{|\vec{b}|} = \vec{a}.\ \hat{b}$

99 Projection of $\vec{b}$ on $\vec{a} = \dfrac{\vec{a}\,.\vec{b}}{|\vec{a}|} = \vec{b}.\ \hat{a}$

100 $\vec{a}.\vec{b} = 0 \Leftrightarrow \vec{a}$ is perpendicular to $\vec{b}$

101 $\vec{a}.\vec{b} = \vec{b}.\vec{a}$

102 $\vec{a}.\vec{a} = |\vec{a}|^2$

103 $m\vec{a}.\vec{b} = m(\vec{a}.\vec{b}) = \vec{a}.m\vec{b}$, for any scalar m

104 $m\vec{a}.n\vec{b} = mn(\vec{a}.\vec{b}) = \vec{a}.mn\vec{b}$, for scalars m ,n

105 $|\vec{a}+\vec{b}| \leq |\vec{a}| + |\vec{b}|$

106 $|\vec{a}-\vec{b}| \geq |\vec{a}| - |\vec{b}|$

107 $|\vec{a}\pm\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 \pm 2(\vec{a}.\vec{b})$

108 $(\vec{a}+\vec{b})(\vec{a}-\vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$

109 $\vec{a}.\vec{b} > 0$, iff θ is acute.

110 $\vec{a}.\vec{b} < 0$, iff θ is obtuse.

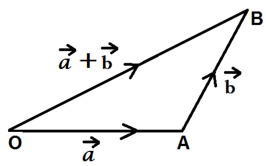
111 If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors, then

$$|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$$

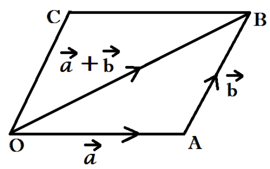
112 If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then

$$\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3.$$

113 *Triangle law of Addition for Vectors* – In a ΔOAB, if $\overrightarrow{OA}$ and $\overrightarrow{AB}$ represent $\vec{a}$ and $\vec{b}$ respectively, then $\overrightarrow{OB}$ represents $(\vec{a}+\vec{b})$.



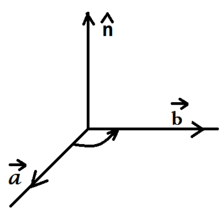
114 *Parallelogram Law of addition for vectors* – In a parallelogram OABC, if $\overrightarrow{OA}$ and $\overrightarrow{AB}$ represent $\vec{a}$ and $\vec{b}$ respectively, then $\overrightarrow{OB}$ represents $(\vec{a}+\vec{b})$.



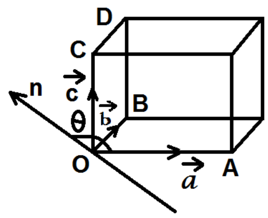
115 *Vector Product of two vectors* – Let $\vec{a}$ and $\vec{b}$ be two nonzero, nonparallel vectors and let θ be the angle between them such that $0 < \theta < \pi$. Then, the vector product of $\vec{a}$ and $\vec{b}$ is defined as

$$\vec{a}\times\vec{b} = (|\vec{a}|\,|\vec{b}|\sin\theta)\,\hat{n} \Rightarrow \theta = \sin^{-1}\left\{\frac{|\vec{a}\times\vec{b}|}{|\vec{a}|\,|\vec{b}|}\right\},$$

Where $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$, such that $\vec{a}$,$\vec{b}$, $\hat{n}$ form a right-handed system.



116 *Volume of Parallelopiped* – If $\vec{a}$, $\vec{b}$, $\vec{c}$ be three vectors then the scalar product of $\vec{a}$ with the vector product of $\vec{b}$ and $\vec{c}$ is called the scalar triple product of the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$. It is written as



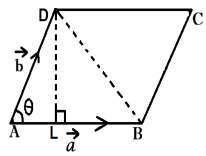
$$V = \vec{a}.(\vec{b} \times \vec{c}) = [\vec{a}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{c}\ \vec{a}] = [\vec{c}\ \vec{a}\ \vec{b}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

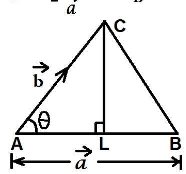
117 Vector Triple Product – If $\vec{a}$, $\vec{b}$, $\vec{c}$ be three vectors then the vector product of $\vec{a}$ with $(\vec{b} \times \vec{c})$, is called the vector triple product and is written as

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}\,.\vec{c})\,\vec{b} - (\vec{a}\,.\vec{b})\,\vec{c}$$

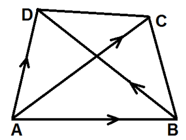
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_2c_3 - b_3c_2 & b_3c_1 - b_1c_3 & b_1c_2 - b_2c_1 \end{vmatrix}$$

118 (a) $ar\,(\| \, gm\ ABCD) = |\vec{a} \times \vec{b}|$, where $\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{AD} = \vec{b}$.





(b) $ar\,(\Delta\,ABC) = \frac{1}{2}|\vec{a} \times \vec{b}|$, where $\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{AC} = \vec{b}$.



(c) $ar(quad.ABCD) = \frac{1}{2}\,|\,\overrightarrow{AC} \times \overrightarrow{BD}\,|$, where AC and BD are its diagonals.

119 Co-linearity – The points A, B, C with position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ are collinear if and only if

$$(\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a}) + (\vec{a} \times \vec{b}) = \vec{0}$$

Area of Triangle – If $\vec{a}$, $\vec{b}$, $\vec{c}$ are the position vectors of the points A, B, C, then *The area of triangle* $= \Delta = \frac{1}{2}\,[(\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a}) + (\vec{a} \times \vec{b})]$

120 Equation of a line passing through two given points –
Vector Form – The vector equation of a line L, passing through two given points A and B with position vectors $\vec{r_1}$ and $\vec{r_2}$, is given by

$$\vec{r} = \vec{r_1} + \lambda\,(\vec{r_2} - \vec{r_1})$$

Cartesian Form – (i) The equations of a line passing through two given points A (x_1, y_1, z_1) and B (x_2, y_2, z_2) are given by

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

(ii) The Cartesian equations of a line with direction ratios *a*, b, c and passing through A (x_1, y_1, z_1) are

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

121 (i) The vector equation of a line through a point with position vector $\overrightarrow{r_1}$ and parallel to $\vec{m}$ is

$$\vec{r} = \overrightarrow{r_1} + \lambda\, \vec{m}$$

(ii) The vector equation of a line through a point with position vectors $\overrightarrow{r_1}$ and $\overrightarrow{r_2}$ is

$$\vec{r} = \overrightarrow{r_1} + \lambda\,(\overrightarrow{r_2} - \overrightarrow{r_1})$$

122 *Cartesian Form* – Three given points A (x_1, y_1, z_1), B (x_2, y_2, z_2) and C (x_3, y_3, z_3) will be collinear if

$$\frac{x_3 - x_1}{x_2 - x_1} = \frac{y_3 - y_1}{y_2 - y_1} = \frac{z_3 - z_1}{z_2 - z_1}$$

Vector Form – Three given points A, B, C with position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ respectively are collinear if $\vec{c} = (1 - \lambda)\,\vec{a} + \lambda\,\vec{b}$

123 *Vector Form* – Let the vector equations of two given lines be $\vec{r} = \overrightarrow{r_1} + \lambda\,\overrightarrow{n_1}$ and $\vec{r} = \overrightarrow{r_2} + \mu\,\overrightarrow{n_2}$, where λ and μ are scalars. Let θ be the angle between these lines. Since the given lines are parallel to $\overrightarrow{n_1}$ and $\overrightarrow{n_2}$ respectively, the angle between the given lines must be equal to the angle between $\overrightarrow{n_1}$ and $\overrightarrow{n_2}$.

$$\cos\theta = \frac{\overrightarrow{n_1} \cdot \overrightarrow{n_2}}{|\overrightarrow{n_1}|\ |\overrightarrow{n_2}|}$$

These planes are parallel, if $\overrightarrow{n_1}$ is parallel to $\overrightarrow{n_2}$.
These planes are perpendicular, if $\overrightarrow{n_1} \cdot \overrightarrow{n_2} = 0$.
Cartesian Form – Let the Cartesian equations of two given lines be

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1} \text{ and } \frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}.$$

Then, the direction ratios of these lines are a_1, b_1, c_1 and a_2, b_2, c_2 respectively. Let θ be angle between these lines. Then

$$\cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\left(\sqrt{a_1^2 + b_1^2 + c_1^2}\right)\left(\sqrt{a_2^2 + b_2^2 + c_2^2}\right)}$$

These planes are parallel, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

These planes are perpendicular, if $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

124 ***Vector Form*** – (i) The shortest distance between two skew lines $\vec{r} = \overrightarrow{a_1} + \lambda\,\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\,\overrightarrow{b_2}$ is given by $d = \left|\frac{(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}\right|$

(ii) Let $L_1\ (\vec{r} = \overrightarrow{a_1} + \lambda\,\overrightarrow{b_1})$ and $L_2\ (\vec{r} = \overrightarrow{a_2} + \mu\,\overrightarrow{b_2})$ be two parallel lines. Then these lines are coplanar if

$$\overrightarrow{a_1} \cdot (\overrightarrow{b_1} \times \overrightarrow{b_2}) = \overrightarrow{a_2} \cdot (\overrightarrow{b_1} \times \overrightarrow{b_2})$$

Distance between Parallel Lines = $|\overrightarrow{BM}| = \left|\frac{\vec{b} \times (\overrightarrow{a_2} - \overrightarrow{a_1})}{|\vec{b}|}\right|$

Condition for two given lines to intersect – Suppose that the lines $\vec{r} = \overrightarrow{a_1} + \lambda\,\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\,\overrightarrow{b_2}$ intersect. Then the shortest distance between them is zero, i.e.

$$[(\overrightarrow{a_2} - \overrightarrow{a_1})\ \overrightarrow{b_1}\ \overrightarrow{b_2}] = 0.$$

Cartesian Form - The shortest distance between two skew lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is given by

$$d = \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{D}}$$

where D = $\{(a_1b_2 - a_2b_1)^2 + (b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2\}$

125 Vector Form – If $\hat{n}$ is a unit vector normal to a given plane, directed from the origin to the plane and p is the length of the perpendicular drawn from the origin to the plane then the vector equation of the plane is $\vec{r}\,.\,\hat{n}$ = p.

A vector normal to the plane $ax + by + cz + d = 0$ is $\vec{n} = a\hat{i} + b\hat{j} + c\hat{k}$.

Cartesian Form – If a, b, c are the direction ratios of the normal to a plane, then the equation of the plane is $ax + by + cz + d = 0$

126 Vector Form – The vector equation of a plane passing through a point having position vector $\vec{a}$ and normal to $\vec{n}$ is

$$(\vec{r} - \vec{a}).\vec{n} = 0 \text{ or } \vec{r}.\vec{n} = \vec{a}.\vec{n}$$

Cartesian Form – The equation of a plane passing through a point P (x_1, y_1, z_1) is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$, where a, b, c are constants.

127 The vector equation of a plane passing through a point having position vector $\vec{a}$ and parallel to vectors $\vec{b}$ and $\vec{c}$ is

$\vec{r} = \vec{a} + m\,\vec{b} + n\,\vec{c}$, where *m* and *n* are parameters.

or $\vec{r}\,.(\vec{b} \times \vec{c}) = \vec{a}\,.(\vec{b} \times \vec{c})$

128 If a plane makes intercepts of lengths *a*, b, c with the *x*-axis, y-axis and z-axis respectively, the equation of the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

129 Vector Form – The equation of a plane through the intersection of two planes $\vec{r}.\vec{n_1} = d_1$ and $\vec{r}.\vec{n_2} = d_2$ is given by

$$\vec{r}.(\vec{n_1} + \lambda\,\vec{n_2}) = d_1 + \lambda\, d_2$$

Cartesian Form – The equation of a plane through the intersection of two planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by $(a_1x + b_1y + c_1z + d_1) + \lambda\,(a_2x + b_2y + c_2z + d_2) = 0$

130 The Cartesian equation of a plane passing through points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is

$$\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0 \text{ or } \begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

131 The equation of a plane parallel to the plane

(a) $\vec{r}\,.\vec{n} = d$ is $\vec{r}\,.\vec{n} = d_1$

(b) $ax + by + cz + d = 0$ is $ax + by + cz + \lambda = 0$

132 The length of perpendicular from the point (x_1, y_1, z_1) to the plane $ax + by + cz + d = 0$ is

$$\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

133 The distance between the parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ is given by

$$\frac{|d_2 - d_1|}{\sqrt{a^2 + b^2 + c^2}}$$

134 The equation of the family of planes containing the lines $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is

$$a_1x + b_1y + c_1z + d_1 + \lambda(a_2x + b_2y + c_2z + d_2) = 0,$$

where λ is a parameter.

135 The equations of the planes bisecting the angles between the planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are given by

$$\frac{a_1x + b_1y + c_1z + d_1}{\sqrt{a^2 + b^2 + c^2}} = \pm \frac{a_2x + b_2y + c_2z + d_2}{\sqrt{a^2 + b^2 + c^2}}$$

136 The angle θ between a line $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ and a plane $ax + by + cz + d = 0$ is the complement of the angle between the line and normal to the plane and is given by

$$\sin\theta = \frac{al+bm+cn}{\sqrt{a^2 + b^2 + c^2}\sqrt{l^2 + m^2 + n^2}}$$

The angle θ between the line $\vec{r} = \vec{a} + \lambda\vec{b}$ and the plane $\vec{r}.\vec{n} = d$ is given by

$$\sin\theta = \frac{\vec{b}.\vec{n}}{|\vec{b}||\vec{n}|}$$

137 Two lines $\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1}$ and $\frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2}$ are coplanar, if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

and the equation of the plane containing them is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0 \quad \text{or} \quad \begin{vmatrix} x - x_2 & y - y_2 & z - z_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

III. Commercial Mathematics

1 $Cost\ Price = Buying\ Price + Overhead\ Expenses$

2 $Profit = Sale\ Price - Cost\ Price$

3 $Loss = Cost\ Price - Sale\ Price$

4 $Profit\ \% = \frac{Profit}{Cost\ Price} \times 100\%$

5 $Loss\ \% = \frac{Loss}{Cost\ Price} \times 100\%$

6 $Sale\ Price = \left(\frac{100\ \% + Profit\ \%}{100\ \%}\right) \times Cost\ Price$

7 $Sale\ Price = \left(\frac{100\ \% - Loss\ \%}{100\ \%}\right) \times Cost\ Price$

8 $Discount = Marked\ Price - Sale\ Price = Discount\ \%\ of\ Marked\ Price$

9 $Sales\ Tax = Tax\ \%\ of\ Bill\ Amount$

10 $Simple\ Interest = \frac{PRT}{100}$

11 $Amount = Principal + Simple\ Interest$

12 *If the Interest is compounded annually, then*

$$A = P\left(1 + \frac{R}{100}\right)^n,$$

where $\boldsymbol{A}$ *is Amount,* $\boldsymbol{P}$ *is Principal,* $\boldsymbol{R}$ *is Rate of Interest and* $\boldsymbol{n}$ *is time period.*

13 *If the Interest is compounded half yearly, then*

$$A = P\left(1 + \frac{R}{200}\right)^{2n},$$

14 $If\ the\ Interest\ is\ compounded\ quarterly, then$

$$A = P\left(1 + \frac{R}{400}\right)^{4n},$$

15 $Compund\ Interest = P\left[\left(1 + \frac{R}{100}\right)^{n} - 1\right]$

16 Speed = $\frac{Distance}{Time}$

IV. Geometry

1 $Sum\ of\ all\ the\ interior\ angles\ of\ a\ polygon\ =\ (n - 2) \times 180°,$
$where\ 'n'\ is\ the\ number\ of\ sides\ of\ the\ polygon$

2 $Measure\ of\ each\ Exterior\ Angle\ = \frac{360°}{n}$

3 $Measure\ of\ each\ Interior\ Angle\ = 180° - \frac{360°}{n} = \frac{(n-2) \times 180°}{n}$

4 The relation between three systems of measurement of an angle is

$$\frac{D}{90°} = \frac{G}{100} = \frac{2R}{\pi}$$

Where the three systems of measuring angles are:

(i) Sexagesimal system, in which we have
1 right angle = 90 degrees (= 90°)
$1° = 60\ minutes\ (= 60')$
$1' = 60\ seconds\ (= 60'')$

(ii) Centesimal system, in which we have
1 right angle = 100 grades
1grade = 100 minutes
1 minute = 100 seconds

(iii) Circular system, in which the unit of measurement is radian. One radian is the measure of an angle subtended at the centre of a circle by an arc of length equal to the radius of the circle.
Π radians = 180°

V. Coordinate Geometry

1 $The\ Distance\ Formula$
$- The\ Distance\ between\ the\ points\ P(x_1, y_1)\ \&\ Q(x_2, y_2)$

$$= PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

2 $The\ Section\ Formula - The\ coordinates\ of\ point\ P(x, y), which\ divides\ the$
$line\ segment$
$joining\ the\ points\ A(x_1, y_1)\ and\ B(x_2, y_2), in\ the\ ratio\ m_1 : m_2\ are$
$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right), internally\ and\ \left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right)$, externally.

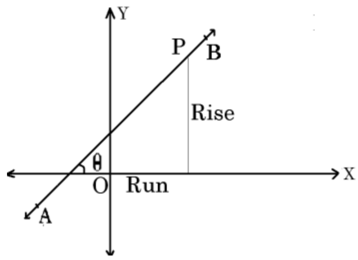
3 $The\ Mid\ point\ Formula$ – The coordinates of the midpoint P of the $A(x_1, y_1)$ and $B(x_2, y_2)$ are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

4 $Area\ of\ a\ \Delta\ ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)],$
$where\ A(x_1, y_1), B(x_2, y_2)\ and\ C(x_3, y_3)\ are\ the\ vertices\ of\ the\ \Delta\ ABC.$

5 $The\ Slope\ of\ a\ line\ AB\ = m == tan\theta = \frac{Rise}{Run}$

$$= \frac{Difference\ of\ Ordinates}{Difference\ of\ abscissae} = \frac{y_2 - y_1}{x_2 - x_1},$$

$Where\ (x_1, y_1)\ and\ (x_2, y_2)\ are\ the\ two\ on\ line\ AB.$



6 An acute angle θ between the lines having slopes m_1 and m_2 is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|,\ \text{where } 1 + m_1 m_2 \neq 0.$$

7 The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

i.e. $PQ = \sqrt{(Difference\ of\ abscissae)^2 + (Difference\ of\ ordinates)^2}$

8 The distance of a point ***P*** $(\boldsymbol{x}, \boldsymbol{y})$ from the origin **O (0, 0)** is given by $OP = \sqrt{x^2 + y^2}$.

9 The area of the triangle, the coordinates of whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, is the absolute value of

$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$ or $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

10 If the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are collinear, then $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

11 The coordinates of the mid-point of the line segment joining (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

12 The coordinates of the centroid of the triangle whose vertices are (x_1, y_1), (x_2, y_2) and(x_3, y_3) are $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

13 ***Slope-Intercept form*** – The equation of a line with slope m and making an intercept ***c*** on y-axis is $y = mx + c$.

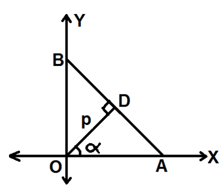
14 ***Point-Slope Form*** – The equation of the line which passes through the point (x_1, y_1) and has slope m is $y - y_1 = m(x - x_1)$

15 The equation of the line which passes through the point (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

16 ***Intercept Form*** – The equation of the line making intercepts a and b on x and y-axis respectively is $\frac{x}{a} + \frac{y}{b} = 1$.

17 Normal form of a straight line $x\ \cos\alpha + y\ \sin\alpha = p$, where p is the length of perpendicular from origin to the straight line and α is the angle made by this perpendicular with the positive direction of *x*-axis.



18 General Equation of Circle – $(x-h)^2+(y-k)^2=r^2$
Where *(h, k)* is the centre of the circle and *r* is radius of the circle.

19 Standard Equation of circle − $x^2+y^2+2gx+2fy+c=0$
Where $(-g,-f)$ is the centre of the circle and $r\left(=\sqrt{g^2+f^2-c}\right)$ is the radius of the circle.

20 If (α,β) is the focus and $ax+by+c=0$ is the equation of the directrix of a parabola, then its equation is $(x-\alpha)^2+(y-\beta)^2=\frac{(ax+by+c)^2}{a^2+b^2}$. This equation is of the form $ax^2+2hxy+by^2+2gx+2fy+c=0$ satisfying the conditions $abc+2fgh-af^2-bg^2-ch^2\neq 0$ and $h^2-ab=0$.

21 Following are four standard forms of parabola:

	$y^2=4ax$	$y^2=-4ax$	$x^2=4ay$	$x^2=-4ay$
Coordinates of vertex	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$
Coordinates of focus	$(a,0)$	$(-a,0)$	$(0,a)$	$(0,-a)$
Equation of the directrix	$x=-a$	$x=a$	$y=-a$	$y=a$
Equation of the axis	$y=0$	$y=0$	$x=0$	$x=0$
Length of the Latus – rectum	$4a$	$4a$	$4a$	$4a$
Focal distance of a point P (x,y)	$a+x$	$a-x$	$a+y$	$a-y$

22 (i) If S is the focus and ZZ' is the directrix and P is any point on the ellipse such that M is the foot of perpendicular from P on ZZ' then SP = e . PM
The equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents an ellipse, if
$\Delta=abc+2fgh-af^2-bg^2-ch^2\neq 0$ and $h^2<ab$.

(ii) If the centre of the elipse is at the point (h,k) and the directions of the axes are parallel to the coordinate axes, then its equation is
$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

(iii) The sum of the focal distances of any point on the elipse
= SP + S 'P = $a-ex+a+ex=2a$ = major axis (= constant),
Where P(*x*, *y*) is any point on the elipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

23 The equation of the ellipse whose axes are parallel to the coordinate axes and whose centre is at the origin, is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with the following properties:

	$\frac{x^2}{y^2}+\frac{y^2}{b^2}=1, a>b$	$\frac{x^2}{y^2}+\frac{y^2}{b^2}=1, a<b$
Coordinates of the centre	(0, 0)	(0, 0)
Coordinates of the vertices	$(a,0)$ and $(-a,0)$	$(0,b)$ and $(0,-b)$
Coordinates of foci	$(ae,0)$ and $(-ae,0)$	$(0,be)$ and $(0,-be)$
Length of the major axis	$2a$	$2b$
Length of the minor axis	$2b$	$2a$
Equation of the major axis	$y=0$	$x=0$
Equation of the minor axis	$x=0$	$y=0$
Equations of the directices	$x=\frac{a}{e}$ and $x=-\frac{a}{e}$	$y=\frac{b}{e}$ and $y=-\frac{b}{e}$
Eccentricity	$e=\sqrt{1-\frac{b^2}{a^2}}$	$e=\sqrt{1-\frac{a^2}{b^2}}$
Length of the latus-rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$
Focal distances of a point (x,y)	$a\pm ex$	$b\pm ey$

24 The equation of the hyperbola having its centre at the origin and axes along the coordinate axes is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with the following properties:

	Hyperbola $\frac{x^2}{y^2} - \frac{y^2}{b^2} = 1$	Conjugate hyperbola $-\frac{x^2}{y^2} + \frac{y^2}{b^2} = 1$
Coordinates of the centre	(0, 0)	(0, 0)
Coordinates of the vertices	$(a, 0)$ and $(-\ a, 0)$	$(0, b)$ and $(0, -b)$
Coordinates of foci	$(\pm\ ae, 0)$	$(0, \pm\ be)$
Length of the transverse axis	$2a$	$2b$
Length of the conjugate axis	$2b$	$2a$
Equation of the transverse axis	$y = 0$	$x = 0$
Equation of the conjugate axis	$x = 0$	$y = 0$
Equations of the direectices	$x = \pm\ \frac{a}{e}$	$y = \pm\ \frac{b}{e}$
Eccentricity	$e = \sqrt{1 + \frac{b^2}{a^2}}$ or $b^2 = a^2(e^2 - 1)$	$e = \sqrt{1 + \frac{a^2}{b^2}}$ or $a^2 = b^2(e^2 - 1)$
Length of the latus-rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$

25 If the centre of the hyperbola is at the point (h, k) and the directions of the axes are parallel to the coordinate axes, then its equation is $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

26 Distance between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

27 Section Formula: If R divides PQ in the ratio $m{:}n$, then

$$R_x = \frac{mQ_x \pm nP_x}{m \pm n}$$

$$R_y = \frac{mQ_y \pm nP_y}{m \pm n}$$

$$R_z = \frac{mQ_z \pm nP_z}{m \pm n}$$

Note:- "+" sign for internal division & "-" sign for external division.

VI. Mensuration

1 $Perimeter\ of\ a\ Polygon\ =\ Sum\ of\ all\ the\ sides\ of\ the\ polygon$

2 $Perimeter\ of\ Rectangle\ =\ 2(length + breadth) = 2(l + b)$

3 $Perimeter\ of\ Square\ =\ 4 \times side = 4a$

4 $Circumference\ of\ a\ Circle\ =\ C\ =\ 2\pi r, where\ \boldsymbol{r}\ is\ the\ radius\ of\ the\ circle$

5 $Perimeter\ of\ a\ Semicircle = \pi r + 2r = (\pi + 2)r$

6 $Area\ of\ Circle\ = \pi r^2$

7 $Area\ of\ a\ Semicircle = \frac{1}{2}\pi r^2$

8 $Length\ of\ an\ Arc = \frac{\theta}{360°} \times 2\pi r$

9 $Area\ of\ a\ Sector = \frac{\theta}{360°} \times \pi r^2$

10 $Heron's\ Formula\ -\ Area\ of\ a\ Scalene\ Triangle\ = \sqrt{s(s-a)(s-b)(s-c)},$
$where\ \boldsymbol{s}\ is\ the\ semi-perimeter\ of\ the\ Scalene\ Triangle,\qquad \boldsymbol{s} = \frac{\mathbf{1}}{\mathbf{2}}(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})$
$and\ \boldsymbol{a}, \boldsymbol{b}\ \&\ \boldsymbol{c}\ are\ the\ sides\ of\ the\ Scalene\ Triangle.$

11 $Area\ of\ a\ Right\ Angled\ Triangle = \frac{1}{2} \times base \times height = \frac{1}{2} \times b \times h$

12 $Area\ of\ an\ Isosceles\ Triangle = \frac{1}{2} \times \mathrm{b} \times \sqrt{\mathrm{a}^2 - \left(\frac{\mathrm{b}}{2}\right)^2}$

Where ‘**b**’ is the base and ‘**a**’ is one of the equal sides of the Isosceles Triangle

13 $Area\ of\ an\ Equilateral\ Triangle = \frac{\sqrt{3}}{4} \times side \times side = \frac{\sqrt{3}}{4}a^2$

14 $Area\ of\ Rhombus$

$$= \frac{1}{2} \times d_1 \times d_2, where\ \boldsymbol{d_1}\ \&\ \boldsymbol{d_2}\ are\ the\ diagonals\ of\ the\ Rhombus$$

15 $Area\ of\ Parallelogram = base \times height = b \times h$

16 $Area\ of\ Rectangle = Length\ \times breadth = l \times b$

17 $Area\ of\ Square = side\ \times side = a^2$

18 $Area\ of\ Trapezium\ or\ Trapezoid = \frac{1}{2} \times sum\ of\ parallel\ sides \times height$

$$= \frac{1}{2} \times (a+b) \times h$$

19 $Euler's\ Formula\qquad \boldsymbol{F} + \boldsymbol{V} - \boldsymbol{E} = \mathbf{2},\ \ where\ ‘\boldsymbol{F}’\ stands\ for\ number\ of\ faces,$
$‘\boldsymbol{V}’\ stands\ for\ number\ of\ vertices\ and\ '\boldsymbol{E}'stands\ for\ number\ of\ edges$

20 $Area\ of\ four\ walls\ of\ a\ recangular\ box\ or\ Curved\ Surface\ Area\ of\ Cuboid$
$= 2(l+b) \times h$

21 $Total\ Surface\ Area\ of\ a\ Cuboid = 2\ (lb + bh + hl)$

22 $Volume\ of\ a\ Cuboid = l \times b \times h$

23 $Curved\ or\ Lateral\ Surface\ Area\ of\ a\ Cube = 4\ l^2$

24 $Total\ Surface\ Area\ of\ a\ Cube = 6\ l^2$

25 $Volume\ of\ a\ Cube = l^3$

26 $Curved\ or\ Lateral\ Surface\ Area\ of\ a\ Cylinder = 2\pi rh$

27 $Total\ Surface\ Area\ of\ a\ Cylinder = 2\pi r(r+h)$

28 $Volume\ of\ a\ Cylinder = \pi r^2 h$

29 $Curved\ or\ Lateral\ Surface\ Area\ of\ a\ Cone = \pi rl,$

$$where\ \boldsymbol{l}\ \left(= \sqrt{r^2+h^2}\right)\ is\ slant\ height\ of\ cone$$

30 $Total\ Surface\ Area\ of\ Cone = \pi r(r + l)$

31 $Volume\ of\ a\ Cone = \frac{1}{3}\pi r^2 h$

32 $Curved\ or\ Lateral\ Surface\ Area\ of\ a\ Prism$
$= height \times Perimeter\ of\ the\ base = h \times P$

33 $Total\ Surface\ Area\ of\ a\ Prism$
$= 2 \times Area\ of\ the\ base + h \times Perimeter\ of\ the\ base = 2A + hP$

34 $Volume\ of\ a\ Prism = Area\ of\ base \times height = Ah$

35 $Curved\ or\ Lateral\ Surface\ Area\ of\ a\ Pyriamid$
$= Number\ of\ sides\ at\ the\ base \times Area\ of\ a\ triangle = n \times \frac{1}{2} \times b \times h$

36 $Total\ Surface\ Area\ of\ a\ Pyramid$
$= Area\ of\ base + Combined\ Area\ of\ the\ lateral\ faces = A + n \times \frac{1}{2} \times b \times h$

37 $Volume\ of\ a\ Pyramid = \frac{1}{3} \times Area\ of\ Base \times height = \frac{1}{3} \times A \times h$

38 $Surface\ Area\ of\ a\ Sphere = 4\pi r^2$

39 $Volume\ of\ a\ Sphere = \frac{4}{3}\pi r^3$

40 $Surface\ Area\ of\ a\ Hemisphere = 3\pi r^2$

41 $Volume\ of\ a\ Hemisphere = \frac{2}{3}\pi r^3$

42 $Total\ Surface\ Area\ of\ a\ Hollow\ Cylinder = 2\pi rh + 2\pi Rh + 2\pi(R^2 - r^2)$

43 $Volume\ of\ a\ Hollow\ Cylinder = \pi(R^2 - r^2)h$

44 $Curved\ Surface\ Area\ of\ Frustum\ of\ a\ Cone\ or\ Truncated\ Cone$
$= \pi l(r_1 + r_2),$
$where\ \boldsymbol{l}\left(= \sqrt{h^2 + (r_1 - r_2)^2}\right)\ is\ slant\ height\ of\ the\ frustum$

45 $Total\ Surface\ Area\ of\ Frustum\ of\ a\ Cone\ or\ Truncated\ Cone$
$= \pi l(r_1 + r_2) + \pi({r_1}^2 + {r_2}^2)$

46 $Volume\ of\ Frustum\ of\ a\ Cone\ or\ Truncated\ Cone$
$= \frac{1}{3}\pi h({r_1}^2 + {r_2}^2 + r_1 r_2),\ \ where\ \boldsymbol{h}\ is\ vertical\ height\ of\ the\ frustum,$
$\boldsymbol{r_1}\ \&\ \boldsymbol{r_2}\ are\ radii\ of\ the\ two\ bases\ (ends) of\ the\ frustum$

VII. Statistics

1 $Mean\ (for\ ungrouped\ data) = \frac{Sum\ of\ all\ the\ observations}{Total\ number\ of\ observations}$

2 $Mean\ (for\ grouped\ data - The\ Direct\ Method) = \overline{x} = \frac{\Sigma f_i x_i}{\Sigma f_i},$
$where\ x_i\ are\ the\ class\ marks$

3 $Mean\ \left(for\ grouped\ data - The\ Assumed\ Mean\ Method\right) = \overline{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i},$
$where\ \boldsymbol{a}\ is\ the\ assumed\ mean, \boldsymbol{d_i}\ is\ the\ deviation\ from\ the\ class\ mark\ and$
$\boldsymbol{d_i = x_i - a}$

4 $Mean\ (for\ grouped\ data - The\ Step\ Deviation\ Method) = \overline{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) h,$
$where\ \boldsymbol{h}\ is\ class\ size\ of\ each\ class\ interval\ and\ \boldsymbol{u_i} = \frac{x_i - a}{h}$

5 $Median\ (for\ ungrouped\ data\ \&\ Odd\ number\ of\ observations)$
$$= \left(\frac{N+1}{2}\right)^{th}\ observation, where\ \boldsymbol{N}\ is\ the\ total\ number\ of\ observations$$

6 $Median\ (for\ ungrouped\ data\ \&\ Even\ number\ of\ observations)$
$$= Mean\ of\ \left(\frac{N}{2}\right)^{th}\ \&\ \left(\frac{N}{2}+1\right)^{th}\ observations$$

7 $Median(for\ grouped\ data) = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h,$
$where\ \boldsymbol{l}\ is\ lower\ limit\ of\ median\ class, \boldsymbol{N}\ is\ total\ number\ of\ observations,$ $\boldsymbol{cf}\ is\ cumulative\ frequency\ of\ class\ preceding\ the\ median\ class, \boldsymbol{f}\ is$ $frequency\ of\ median\ class, \boldsymbol{h}\ is\ class\ size\ (assuming\ class\ size\ to\ be\ equal)$

8 $Mode(for\ ungrouped\ data)$
$$= The\ observation\ which\ occurs\ most\ number\ of\ times$$

9 $Mode(for\ grouped\ data) = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h,$
$where\ \boldsymbol{l}\ is\ lower\ limit\ of\ the\ modal\ class, \boldsymbol{h}\ is\ size\ of\ the\ class\ interval$ $(assuming\ all\ class\ sizes\ to\ be\ equal), \boldsymbol{f_1}\ is\ frequency\ of\ the\ modal\ class,$
$\boldsymbol{f_0}\ is\ frequency\ of\ the\ class\ preceding\ the\ modal\ class,$
$\boldsymbol{f_2}\ is\ frequency\ of\ the\ class\ succeeding\ the\ modal\ class$

10 Theoretical Probability = $\frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$

11 Experimental Probability = $\frac{Number\ of\ favourable\ trials}{Total\ number\ of\ trials}$

12 Probability (Multiplication Rule) – For two independent events E_1 and E_2

(i) $P(E_1 \cap E_2) = P(E_1) \times P(E_2)$
(ii) $P(E_1 \cap \overline{E_2}) = P(E_1) \times P(\overline{E_2})$
(iii) $P(\overline{E_1} \cap E_2) = P(\overline{E_1}) \times P(E_2)$
(iv) $P(\overline{E_1} \cap \overline{E_2}) = P(\overline{E_1}) \times P(\overline{E_2})$

13 Conditional Probability – Probability of A when B has already occurred

(i) $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{n\ (A \cap B)}{n(B)}, P(B) \neq 0$
(ii) $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{n\ (A \cap B)}{n(A)}, P(A) \neq 0$

14 Theorem of Total Probability: - Let $E_1, E_2, \dots\dots\dots, E_n$ be mutually exclusive and exhaustive events associated with a random experiment and let E be an event occurs with some E_i, then

$$P(E) = \sum_{i=1}^{n} P\left(\frac{E}{E_i}\right) . P(E_i)$$

15 Bayes' Theorem – Let $E_1, E_2, \ldots\ldots, E_n$ be mutually exclusive & exhaustive events, associated with a random experiment and let E be any event that occurs with some E_i then

$$P\left(\frac{E_i}{E}\right) = \frac{P\left(\frac{E}{E_i}\right).P(E_i)}{\sum_{i=1}^{n} P\left(\frac{E}{E_i}\right)P(E_i)}$$

16 Probability distribution of X is given by

X	x_1	x_2	……………………..	x_n
P(X)	P_1	P_2	……………………..	P_n

$$Where\ each\ P_i \geq 0\ \&\ \sum_{i=1}^{n} P_i = 1$$

$$Mean\ \ \mu = E(X) = \sum_{i=1}^{n} x_i P_i$$

$$Variance\ \sigma^2 = \left(\sum x_i{}^2 P_i - \mu^2\right)$$

$$Standard\ Deviation\ \ \sigma = \sqrt{Variance}$$

17 Binomial Distribution: $P(X = r) = {}^{n}c_r\, p^r q^r$, where p & q are probability of success & failure and $p + q = 1$.

$$Mean\ \ \mu = \sum_{i=1}^{n} x_i P_i = np$$

$$Variance\ \sigma^2 = \left(\sum x_i{}^2 P_i - \mu^2\right) = npq$$

$$Standard\ Deviation\ \ \sigma = \sqrt{Variance} = \sqrt{npq}$$

18 Mean deviation is the arithmetic mean of the absolute values of deviations about some point (mean or median or mode).

(i) For individual observation, we have

$$M.D. = \frac{1}{n}\sum_{i=1}^{n} |x_i - a|,$$

where a = mean, median, mode

Also, $M.D. = a + h\left\{\frac{1}{N}\sum_{i=1}^{n}|u_i|\right\}$, where $u_i = \dfrac{x_i - a}{h}$.

(ii) For a discrete frequency distribution, we have

$M.D. = \frac{1}{N}\sum_{i=1}^{n} f_i |x_i - a|$, a = mean, median, mode

$M.D. = a + h\left\{\frac{1}{N}\sum_{i=1}^{n} f_i u_i\right\}$, where $u_i = \frac{x_i - a}{h}$.

19 Variance is the arithmetic mean of the squares of deviations about mean $\overline{x}$

(i) For individual observations, we have

$$Variance = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \overline{X}\right)^2$$

Also, $Var\ (X) = \frac{1}{n}\sum_{i=1}^{n}{x_i}^2 - \left(\frac{1}{n}\sum_{i=1}^{n}x_i\right)^2$

$$Var\ (X) = h^2\left[\frac{1}{n}\sum_{i=1}^{n}{u_i}^2 - \left(\frac{1}{n}\sum_{i=1}^{n}u_i\right)^2\right],\text{ where } u_i = \frac{x_i - a}{h}.$$

(ii) For a discrete frequency distribution, we have

$$Var\ (X) = \frac{1}{N}\sum_{i=1}^{n}f_i\left(x_i - \overline{X}\right)^2$$

$$\Rightarrow Var\ (X) = \frac{1}{N}\sum_{i=1}^{n}f_i{x_i}^2 - \left(\frac{1}{N}\sum_{i=1}^{n}f_ix_i\right)^2$$

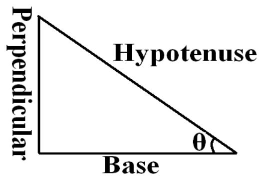
$$\Rightarrow Var\ (X) = h^2\left[\frac{1}{N}\sum_{i=1}^{n}f_i{u_i}^2 - \left(\frac{1}{N}\sum_{i=1}^{n}f_iu_i\right)^2\right]$$

20 The coefficient of variation = C. V. = $\frac{\sigma}{\overline{X}} \times 100$

VIII. Trigonometry and Inverse Trigonometry

Standard Angles → / T. Ratios ↓	**0°**	**30°**	**45°**	**60°**	**90°**
sin *θ*	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos *θ*	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan *θ*	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not Defined
cot *θ*	Not Defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec *θ*	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not Defined
cosec *θ*	Not Defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

1 $\sin\theta = \frac{Perpendicular}{Hypotenuse}$ 2 $\text{cosec}\theta = \frac{Hypotenuse}{Perpendicular}$



3 $\cos\theta = \frac{Base}{Hypotenuse}$ 4 $\sec\theta = \frac{Hypotenuse}{Base}$

5 $\tan\theta = \frac{Perpendicular}{Base}$ 6 $\cot\theta = \frac{Base}{Perpendicular}$

7 $\sin\theta = \frac{1}{\text{cosec}\theta}$ 8 $\text{cosec}\theta = \frac{1}{\sin\theta}$

9 $\cos\theta = \frac{1}{\sec\theta}$ 10 $\sec\theta = \frac{1}{\cos\theta}$ 11 $\tan\theta = \frac{1}{\cot\theta}$

12 $\cot\theta = \frac{1}{\tan\theta}$ 13 $\tan\theta = \frac{\sin\theta}{\cos\theta}$ 14 $\cot\theta = \frac{\cos\theta}{\sin\theta}$

15 $\sin^2\theta + \cos^2\theta = 1$ **Or** $\sin^2\theta = 1 - \cos^2\theta$ **Or** $\cos^2\theta = 1 - \sin^2\theta$

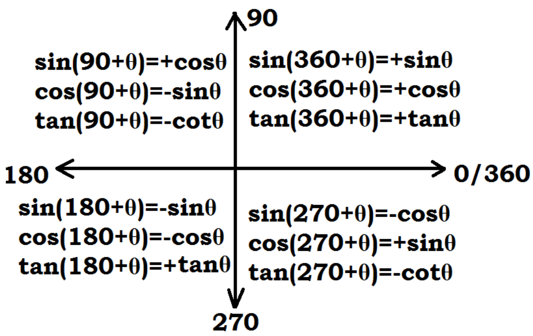
16 $1 + \tan^2\theta = \sec^2\theta$ **Or** $\tan^2\theta = \sec^2\theta - 1$ **Or** $\sec^2\theta - \tan^2\theta = 1$

Or $\sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta}$

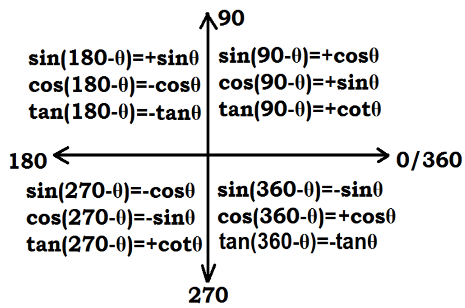
17 $1 + \cot^2\theta = \text{cosec}^2\theta$ **Or** $\cot^2\theta = \text{cosec}^2\theta - 1$ **Or** $\text{cosec}^2\theta - \cot^2\theta = 1$

Or $\text{cosec}\theta - \cot\theta = \frac{1}{\text{cosec}\theta + \cot\theta}$

18



19



20 $\sin(-\theta) = -\sin\theta$ 21 $\cos(-\theta) = \cos\theta$

22 $\tan(-\theta) = -\tan\theta$ 23 $\text{cosec}(-\theta) = -\text{cosec}\theta$

24 $\sec(-\theta) = \sec\theta$ 25 $\cot(-\theta) = -\cot\theta$

26 $\text{Sin}(A + B) = \sin A\cos B + \cos A\sin B$ 27 $\text{Sin}(A - B) = \sin A\cos B - \cos A\sin B$

28 $\cos(A + B) = \cos A\cos B - \sin A\sin B$ 29 $\cos(A - B) = \cos A\cos B + \sin A\sin B$

30 $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$ 31 $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

32 $Sin(A + B) + sin(A - B) = 2sinAcosB$

33 $Sin(A + B) - sin(A - B) = 2cosAsinB$

34 $cos(A + B) + cos(A - B) = 2cosAcosB$

35 $cos(A - B) - cos(A + B) = 2\ sinAsinB$

36 $sin(A + B)sin(A - B) = sin2A - sin2B$

37 $cos(A + B)cos(A - B) = cos2A - sin2B$

38 $sinC + sinD = 2sin\left(\frac{C+D}{2}\right)cos\left(\frac{C-D}{2}\right)$

39 $sinC - sinD = 2cos\left(\frac{C+D}{2}\right)sin\left(\frac{C-D}{2}\right)$

40 $cosC + cosD = 2cos\left(\frac{C+D}{2}\right)cos\left(\frac{C-D}{2}\right)$

41 $cosC - cosD = 2sin\left(\frac{C+D}{2}\right)sin\left(\frac{D-C}{2}\right)$

42 $sin2\theta = 2sin\theta cos\theta = \frac{2tan\theta}{1+tan^2\theta}$

43 $cos2\theta = cos2\theta - sin2\theta = 2cos2\theta - 1 = 1 - 2sin2\theta = \frac{1-tan^2\theta}{1+tan^2\theta}$

44 $\tan 2\theta = \frac{2tan\theta}{1-tan^2\theta}$

45 $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

46 $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

47 $\tan 3\theta = \frac{3tan\theta - \tan^3\theta}{1-3\,tan^2\theta}$

48 $1 + \cos 2\theta = 2\cos^2\theta$

OR

$\cos\theta = \sqrt{\frac{1+cos2\theta}{2}}$

49 $1 - \cos 2\theta = 2\sin^2\theta$

OR

$\sin\theta = \sqrt{\frac{1-cos2\theta}{2}}$

50 $\sin 18° = \frac{\sqrt{5}-1}{4}$

51 $\cos 36° = \frac{\sqrt{5}+1}{4}$

52 sin(A + B + C) = sinAcosBcosC + cosAsinBcosC + cosAcosBsinC - sinAsinBsinC

53 cos(A + B + C) = cosAcosBcosC − cosAsinBsinC − sinAcosBsinC − sinAsinBcosC

54 $tan(A + B + C) = \frac{\tan A + \tan B + \tan C - tanAtanBtanC}{1 - tanAtanB - tanBtanC - tanCtanA}$

55 $\sin^{-1}(sin\,\theta) = \theta$, for all $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

56 $\cos^{-1}(\cos\theta) = \theta$, for all $\theta \in [0, \pi]$

57 $\tan^{-1}(\tan\theta) = \theta$, for all $\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$

58 $cosec^{-1}(cosec\,\theta) = \theta$, for all $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\theta \neq 0$

59 $sec^{-1}(\sec\theta) = \theta$, for all $\theta \in [0, \pi]$, $\theta \neq \frac{\pi}{2}$

60 $cot^{-1}(\cot\theta) = \theta$, for all $\theta \in (0, \pi)$

61 $sin(sin^{-1} x) = x$, for all $x \in [-1, 1]$

62 $\cos(\cos^{-1} x) = x$, for all $x \in [-1, 1]$

63 $\tan(\tan^{-1} x) = x$, for all $x \in$ R

64 cosec $(cosec^{-1} x) = x$, for all $x \in (-\infty, -1] \cup [1, \infty)$

65 $\sec(sec^{-1}x) = x$, for all $x \in (-\infty, -1] \cup [1, \infty)$

66 $\cot(\cot^{-1}x) = x$, for all $x \in$ R

67 $\sin^{-1}(-x) = -\sin^{-1}x$, for all $x \in [-1, 1]$

68 $\cos^{-1}(-x) = \pi - \cos^{-1}x$, for all $x \in [-1, 1]$

69 $\tan^{-1}(-x) = -\tan^{-1}x$, for all $x \in$ R

70 $cosec^{-1}(-x) = -cosec^{-1}x$, for all $x \in (-\infty, -1] \cup [1, \infty)$

71 $\sec^{-1}(-x) = \pi - \sec^{-1}x$, for all $x \in (-\infty, -1] \cup [1, \infty)$

72 $\cot^{-1}(-x) = \pi - \cot^{-1}x$, for all $x \in$ R

73 $\sin^{-1}\left(\frac{1}{x}\right) = cosec^{-1}x$, for all $x \in (-\infty, -1] \cup [1, \infty)$

74 $\cos^{-1}\left(\frac{1}{x}\right) = sec^{-1}x$, for all $x \in (-\infty, -1] \cup [1, \infty)$

75 $tan^{-1}\left(\frac{1}{x}\right) = \begin{cases} cot^{-1}x, & for\ x > 0 \\ -\pi + \cot^{-1}x, & for\ x < 0 \end{cases}$

76 $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$, for all $x \in [-1, 1]$

77 $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$, for all $x \in$ R

78 $\sec^{-1}x + \text{cosec}^{-1}x = \frac{\pi}{2}$, for all $x \in (-\infty, -1] \cup [1, \infty)$

79

$$\tan^{-1}x + \tan^{-1}y = \begin{cases} \tan^{-1}\left(\frac{x+y}{1-xy}\right), & if\ xy < 1 \\ \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right), & if\ x > 0,\ y > 0\ and\ xy > 1 \\ -\pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right), & if\ x < 0,\ y < 0\ and\ xy > 1 \end{cases}$$

80

$$\tan^{-1}x - \tan^{-1}y = \begin{cases} \tan^{-1}\left(\frac{x-y}{1+xy}\right), & if\ xy > -1 \\ \pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right), & if\ x > 0,\ y < 0\ and\ xy < -1 \\ -\pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right), & if\ x < 0,\ y > 0\ and\ xy < -1 \end{cases}$$

81 If $x_1, x_2, x_3, \ldots\ldots\ldots, x_n \in$ R, then

$$\tan^{-1}x_1 + \tan^{-1}x_2 + \ldots\ldots + \tan^{-1}x_n = \left(\frac{S_1 - S_3 + S_5 - S_7 + \cdots\ldots}{1 - S_2 + S_4 - S_6 + S_8 + \cdots\ldots}\right),$$

where S_k denotes the sum of the products of $x_1, x_2, x_3, \ldots\ldots\ldots, x_n$ taken k at a time.

82 $\sin^{-1}x + \sin^{-1}y$

$$= \begin{cases} \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right), & if\ -1 \le x, y \le 1\ and\ x^2 + y^2 \le 1 \\ & or \\ & if\ xy < 0\ and\ x^2 + y^2 > 1 \\ \pi - \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2}), & if\ 0 < x, y \le 1 and\ x^2 + y^2 > 1 \\ -\pi - \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2}), & if\ -1 \le x, y < 0\ and\ x^2 + y^2 > 1 \end{cases}$$

83 $\sin^{-1}x - \sin^{-1}y$

$$=\begin{cases} \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right), & if -1 \le x, y \le 1 \; and \; x^2+y^2 \le 1 \\ & or \\ & if \; xy > 0 \; and \; x^2+y^2 > 1 \\ \pi - \sin^{-1}(x\sqrt{1-y^2} - y\sqrt{1-x^2}), & if \; 0 < x \le 1, -1 \le y \le 0 \; and \; x^2+y^2 > 1 \\ -\pi - \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2}), & if -1 \le x < 0, 0 < y \le 1 \; and \; x^2+y^2 > 1 \end{cases}$$

84 $\cos^{-1}x + \cos^{-1}y$

$$=\begin{cases} \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right), & if -1 \le x, y \le 1 \; and \; x+y \ge 0 \\ 2\pi - \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right), & if -1 \le x, y \le 1 \; and \; x+y \le 0 \end{cases}$$

85 $\cos^{-1}x - \cos^{-1}y$

$$=\begin{cases} \cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right), & if -1 \le x, y \le 1 \; and \; x \le y \\ -\cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right), & if \;\; 0 \le x \le 1, -1 \le y \le 0 \; and \; x \ge y \end{cases}$$

86

$$2\sin^{-1}x = \begin{cases} \sin^{-1}\left(2x\sqrt{1-x^2}\right), & if \; -\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}} \\ \pi - \sin^{-1}\left(2x\sqrt{1-x^2}\right), & if \; \frac{1}{\sqrt{2}} \le x \le 1 \\ -\pi - \sin^{-1}\left(2x\sqrt{1-x^2}\right), & if -1 \le x \le -\frac{1}{\sqrt{2}} \end{cases}$$

87

$$3\sin^{-1}x = \begin{cases} \sin^{-1}(3x - 4x^3), & if \; -\frac{1}{2} \le x \le \frac{1}{2} \\ \pi - \sin^{-1}(3x - 4x^3), & if \; \frac{1}{2} < x \le 1 \\ -\pi - \sin^{-1}(3x - 4x^3), & if -1 \le x < -\frac{1}{2} \end{cases}$$

88

$$2\cos^{-1}x = \begin{cases} \cos^{-1}(2x^2 - 1), & if \; 0 \le x \le 1 \\ 2\pi - \cos^{-1}(2x^2 - 1), & if -1 \le x \le 0 \end{cases}$$

89

$$3\cos^{-1}x = \begin{cases} \cos^{-1}(4x^3 - 3x), & if \; \frac{1}{2} \le x \le 1 \\ 2\pi - \cos^{-1}(4x^3 - 3x), & if \; -\frac{1}{2} \le x \le \frac{1}{2} \\ 2\pi + \cos^{-1}(4x^3 - 3x), & if \; -1 \le x \le -\frac{1}{2} \end{cases}$$

90

$$2\tan^{-1}x = \begin{cases} \tan^{-1}\left(\frac{2x}{1-x^2}\right), & if \; -1 < x < 1 \\ \pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right), & if \; x > 1 \\ -\pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right), & if \; x < -1 \end{cases}$$

91

$$3\tan^{-1}x = \begin{cases} \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right), & if \;\; -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}} \\ \pi + \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right), & if \;\; x > \frac{1}{\sqrt{3}} \\ -\pi + \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right), & if \;\; x < -\frac{1}{\sqrt{3}} \end{cases}$$

92

$$2\tan^{-1}x = \begin{cases} \sin^{-1}\left(\frac{2x}{1+x^2}\right), & if \; -1 \le x \le 1 \\ \pi - sin^{-1}\left(\frac{2x}{1+x^2}\right), & if \; x > 1 \\ -\pi - \sin^{-1}\left(\frac{2x}{1+x^2}\right), & if \; x < -1 \end{cases}$$

93 $2\tan^{-1}x = \begin{cases} \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), & if\ 0 \le x < \infty \\ -\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), & if\ -\infty < x \le 0 \end{cases}$

94 $\sin^{-1}x = \cos^{-1}\left(\sqrt{1-x^2}\right) = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) = \cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

$= \text{cosec}^{-1}\left(\frac{1}{x}\right)$

95 $\cos^{-1}x = \sin^{-1}\left(\sqrt{1-x^2}\right) = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) = \cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) = \sec^{-1}\left(\frac{1}{x}\right)$

$= \text{cosec}^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

96 $\tan^{-1}x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) = \cot^{-1}\left(\frac{1}{x}\right) = \sec^{-1}\left(\sqrt{1+x^2}\right)$

$= \text{cosec}^{-1}\left(\frac{\sqrt{1+x^2}}{x}\right)$

IX. Calculus

1 $\lim_{x\to a} f(x)$ exists $\Leftrightarrow \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$

2 Let $\lim_{x\to a} f(x) = l$ and $\lim_{x\to a} g(x) = m$. If l and m both exist, then

(i) $\lim_{x\to a} kf(x) = k\ \lim_{x\to a} f(x)$

(ii) $\lim_{x\to a}(f \pm g)(x) = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x) = l \pm m$

(iii) $\lim_{x\to a}(fg)(x) = \lim_{x\to a} f(x)\lim_{x\to a} g(x) = lm$

(iv) $\lim_{x\to a}\left(\frac{f}{g}\right)(x) = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \frac{l}{m}$, $m \neq 0$.

(v) $\lim_{x\to a}\{f(x)\}^{g(x)} = l^m$

3. $\lim_{x\to a}\frac{x^n - a^n}{x-a} = n\,a^{n-1}$

4. $\lim_{x\to 0}\frac{sinx}{x} = 1$

5. $\lim_{x\to 0}\frac{tanx}{x} = 1$

6. $\lim_{x\to a}\frac{\sin(x-a)}{x-a} = 1$

7. $\lim_{x\to a}\frac{\tan(x-a)}{x-a} = 1$

8. $\lim_{x\to 0}\frac{\log(1+x)}{x} = 1$

9. $\lim_{x\to 0}\frac{a^x - 1}{x} = \log_e a, \qquad a > 0$

10. $\lim_{x\to 0}\frac{e^x - 1}{x} = 1$

11. $\frac{d}{dx}(x^n) = nx^{n-1}$

12. $\frac{d}{dx}(x) = 1$

13. $\frac{d}{dx}(e^x) = e^x$

14. $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

15. $\frac{d}{dx}(sin\ x) = \cos x$

16. $\frac{d}{dx}(cos\ x) = -\sin x$

17. $\frac{d}{dx}(tan\ x) = \sec^2 x$

18. $\frac{d}{dx}(cot\ x) = -cosec^2\,x$

19. $\frac{d}{dx}(sec\ x) = \sec x\tan x$

20. $\frac{d}{dx}(cosec\ x) = -\,\text{cosec}\,x\cot x$

21. $\frac{d}{dx}(a^x) = a^x \log_e a, a > 0\ a \neq 1$

22. $\frac{d}{dx}(c) = 0, where\ c\ is\ a\ constant\ function$

23. $\frac{d}{dx}\{f(x) \pm\ g(x)\} = \frac{d}{dx}\ f(x) \pm\ \frac{d}{dx}g(x)$

24 $\frac{d}{dx}\{f(x) \times\ g(x)\} = g(x)\frac{d}{dx}\ f(x) + f(x)\ \frac{d}{dx}g(x)$

25 $\frac{d}{dx}\left\{\frac{f(x)}{g(x)}\right\} = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{\{g(x)\}^2}$

26. A real valued function $f(x)$ defined on $(a,\ b)$ is said to be differentiable at $x = c \in (a,\ b)$, iff $\lim_{x \to c} \frac{f(x)- f(c)}{x-c}\ exists\ finitely$

$$\Leftrightarrow \lim_{x \to c^-} \frac{f(x) - f(c)}{x - c} = \lim_{x \to c^+} \frac{f(x) - f(c)}{x - c}$$

$$\Leftrightarrow \lim_{h \to 0^-} \frac{f(c-h) - f(c)}{-h} = \lim_{h \to 0^+} \frac{f(c+h) - f(c)}{h}$$

$$\Leftrightarrow (LHD\ at\ x = c) = (RHD\ at\ x = c)$$

27 *Rolle's Theorem* – Let *f* be a real valued function defined on the closed interval $[a, b]$ such that

(i) It is continuous on the closed interval $[a, b]$,
(ii) It is differentiable on the open interval (a, b).
(iii) $f\ (a) = f\ (b)$.

Then, there exists at least one real number $c \in (a, b)$ such that $f\ '(c) = 0$.

28 *Lagrange's Mean Value Theorem* – Let *f* be a function defined on $[a, b]$ such that

(i) It is continuous on $[a, b]$,
(ii) It is differentiable on (a, b).

Then, there exists at least one real number $c \in (a, b)$ such that

$$f\ '(c) = \frac{f(b)\ - f(a)}{b\ -\ a}$$

29. If $P(x_1, y_1)$ is a point on the curve $y = f(x)$, then

$y - y_1 = \left(\frac{dy}{dx}\right)_P (x - x_1)$ is the equation of tangent at P.

$y - y_1 = -\ \frac{1}{\left(\frac{dy}{dx}\right)_P}(x - x_1)$ is the equation of the normal at P.

30. The angle between the tangents to two given curves at their point of intersection is defined as the angle of intersection of two curves. If C_1 and C_2 are two curves having equations $y\ =\ f(x)$ and $y =\ g(x)$ respectively such that they intersect at point P. The angle θ of intersection of these two curves is given by

$$\tan\theta\ = \frac{\left(\frac{dy}{dx}\right)_{C_1} \sim \left(\frac{dy}{dx}\right)_{C_2}}{1+\left(\frac{dy}{dx}\right)_{C_1}\left(\frac{dy}{dx}\right)_{C_2}}$$

If the angle of intersection of two curves is a right angle, then the curves are said to intersect orthogonally. The condition for orthogonality of two curves C_1 and C_2 is

$$\left(\frac{dy}{dx}\right)_{C_1} \times \left(\frac{dy}{dx}\right)_{C_2} = -1$$

31. Two curves $ax^2 + by^2 = 1$ and $a'x^2 + b'y^2 = 1$ will intersect orthogonally, if

$$\frac{1}{a} - \frac{1}{b} = \frac{1}{a'} - \frac{1}{b'}$$

32. $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \qquad n \neq -1$

33. $\int \frac{1}{x} \, dx = log_e x + C$

34. $\int a^x \, dx = \frac{a^x}{log_e a} + C, \qquad a \neq -1, a > 0$

35. $\int e^x \, dx = e^x + C$

36. $\int \sin x \, dx = -\cos x + C$

37. $\int \cos x \, dx = \sin x + C$

38. $\int \sec^2 x \, dx = \tan x + C$

39. $\int \text{cosec}^2 x \, dx = -\cot x + C$

40. $\int \sec x \tan x \, dx = \sec x + C$

41. $\int \text{cosec}\, x \cot x \, dx = -\text{cosec}\, x + C$

42. $\int tan\, x \, dx = log\ |sec\, x| + C$

$= -\log\cos x + C$

43. $\int cot\, x \, dx = log\ |sin\, x| + C$

44. $\int sec\, x \, dx = log\ |sec\, x + tan\, x| + C = log\left|tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + C$

45. $\int cosec\, x \, dx = log\ |cosec\, x - cot\, x| + C = log\left|tan\frac{x}{2}\right| + C$

46. $\int (ax+b)^n \, dx = \frac{1}{a}\frac{(ax+b)^{n+1}}{n+1} + C$

47. $\int \frac{1}{ax+b} \, dx = \frac{1}{a} log_e\ |ax+b| + C$

48. $\int a^{bx+c} \, dx = \frac{1}{b}\frac{a^{bx+c}}{log_e a} + C, \qquad b > 1$

49. $\int e^{bx+c} \, dx = \frac{1}{b}\, e^{bx+c} + C$

50. $\int \sin(ax+b) \, dx = -\frac{1}{a}\cos(ax+b) + C$

51. $\int \cos(ax+b) \, dx = \frac{1}{a}\sin(ax+b) + C$

52. $\int \tan(ax+b) \, dx = \frac{1}{a}\, log\ |sec\,(ax+b)| + C$

53. $\int \cot(ax+b) \, dx = \frac{1}{a}\, log\ |sin\,(ax+b)| + C$

54. $\int \sec^2(ax+b) \, dx = \frac{1}{a}\tan(ax+b) + C$

55. $\int \text{cosec}^2(ax+b) \, dx = -\frac{1}{a}\cot(ax+b) + C$

56. $\int \sec(ax+b)\tan(ax+b) \, dx = \frac{1}{a}\sec(ax+b) + C$

57. $\int \text{cosec } (ax+b) \cot (ax+b)\ dx = -\frac{1}{a} \text{cosec } (ax+b) + C$

58. $\int \sec (ax+b)\ dx = \frac{1}{a}\ log\ |sec(ax+b) + tan\ (ax+b)| + C$

59. $\int \text{cosec } (ax+b)\ dx = \frac{1}{a} \log |\text{cosec}(ax+b) - \cot (ax+b)| + C$

60. $\int \frac{1}{a^2 + x^2}\ dx = \frac{1}{a} \tan^{-1}\frac{x}{a} + C$

61. $\int \frac{1}{x^2 - a^2}\ dx = \frac{1}{2a}\ log \left|\frac{x-a}{x+a}\right| + C$

62. $\int \frac{1}{a^2 - x^2}\ dx = \frac{1}{2a}\ log \left|\frac{a+x}{a-x}\right| + C$

63. $\int \frac{1}{\sqrt{a^2 - x^2}}\ dx = \sin^{-1}\frac{x}{a} + C$

64. $\int \frac{1}{\sqrt{a^2 + x^2}}\ dx = log\ \left|x + \sqrt{a^2 + x^2}\right| + C$

65. $\int \frac{1}{\sqrt{x^2 - a^2}}\ dx = log\ \left|x + \sqrt{x^2 - a^2}\right| + C$

66. $\int \sqrt{a^2 - x^2}\ dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{1}{2} a^2 \sin^{-1}\frac{x}{a} + C$

67. $\int \sqrt{x^2 - a^2}\ dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{1}{2} a^2\ log\ \left|x + \sqrt{x^2 - a^2}\right| + C$

68. $\int \sqrt{x^2 + a^2}\ dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{1}{2} a^2\ log\ \left|x + \sqrt{x^2 + a^2}\right| + C$

69. If u and v are two functions of x, then

$$\int uv\ dx = u(\int v\ dx) - \int \left\{\frac{du}{dx} \int v\ dx\right\} dx$$

70. $\int e^x\{f(x) + f'(x)\}\, dx = e^x f(x) + C$

71. $\int e^{kx}\{k f(x) + f'(x)\}\, dx = e^{kx} f(x) + C$

72. $\int e^{ax} \sin(bx+c)\ dx = \frac{e^{ax}}{a^2 + b^2}\{a \sin(bx+c) - b \cos (bx+c)\} + \lambda$

73. $\int e^{ax} \cos (bx+c)\ dx = \frac{e^{ax}}{a^2 + b^2}\{a \cos (bx+c) + b \sin(bx+c)\} + \lambda$

74 $\int \frac{a \cos x + b \sin x}{c \cos x + d \sin x}\ dx = M.\log(c \cos x + d \sin x) + N.x + C$

where $N^r = M\frac{d}{dx}(D^r) + N.D^r$

75 $\int \frac{a \cos x + b \sin x + c}{d \cos x + e \sin x + f}\ dx$

$$= M.\log(d \cos x + e \sin x) + N.x + \int \frac{dx}{d \cos x + e \sin x + f}$$

where $N^r = M\frac{d}{dx}(D^r) + N.D^r + P$

76 $\int \frac{x^2+1}{x^4+1}\,dx$ OR $\int \frac{1}{x^4+x^2+1}\,dx$ OR $\int \frac{x^2}{x^4+x^2+1}\,dx$

$\Leftrightarrow k\int \frac{1 \mp \frac{1}{x^2}}{\left(x \pm \frac{1}{x}\right)^2 \mp 2}\,dx \Leftrightarrow k\int \frac{dt}{t^2 \mp (\sqrt{2})^2}$

77 $\int \frac{dx}{P\sqrt{Q}}$ (i) If P & Q both are linear, put Q = t^2

(ii) If P is quadratic & Q is linear, put Q = t^2

(iii) If Q is quadratic, P is linear, put P = $\frac{1}{t}$

(iv) If P & Q are both pure quadratic put $x = \frac{1}{t}$.

78. $\int_a^b f(x)dx = \int_a^b f(t)dt$, i.e. integration is independent of the change of variable.

79. $\int_a^b f(x)dx = -\int_b^a f(x)dx$, i.e. if the limits of a definite integral are interchanged, then its value changes by minus sign only.

80. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, where $a < c < b$.

The above property can be generalized into the following form

$$\int_a^b f(x)dx = \int_a^{c_1} f(x)dx + \int_{c_1}^{c_2} f(x)dx + \ldots\ldots + \int_{c_n}^b f(x)dx$$

Where $a < c_1 < c_2 < c_3 \ldots\ldots < c_{n-1} < c_n < b$.

81. $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

82. $\int_{-a}^a f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & if\ f(x)\ is\ an\ even\ function \\ 0, & if\ f(x)\ is\ an\ odd\ function \end{cases}$

83. $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & if\ f(2a-x) = f(x) \\ 0, & if\ f(2a-x) = -f(x) \end{cases}$

84. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

85. (a) If a differential equation is expressible in the form $\frac{dy}{dx} + Py = Q$, where P and Q are functions of *x*, then it is called a linear differential equation.

The solution of this equation is given by

$$y\left(e^{\int P\,dx}\right) = \int\left(Q\,e^{\int P\,dx}\right)dx + C$$

(b) If a differential equation is in the form $\frac{dx}{dy} + Rx = S$, where R and S are functions of *y,* the solution of this equation is given by

$$x\left(e^{\int R\,dy}\right) = \int\left(S\,e^{\int R\,dy}\right)dy + C$$